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THE MATHEMATICS TEACHER

EDITED BY
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ASSOCIATED WITH
EUGENE R. SMITH MAURICE J. BABB

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WHAT SHOULD COMPRISE THE SUBJECT MATTER OF THE ARITHMETIC IN THE ELEMENTARY SCHOOLS AND IN WHAT GRADES SHOULD FORMAL ARITHMETIC BE TAUGHT?

BY H. J. WIGHTMAN.

(*Continued from June Issue.*)

FIFTH GRADE.

Complete Roman notation.

Cancellation emphasized together with complete statement of problems before working.

Complete review of fractions with mastery of division cases.

Explain the "why" in inverting fractional divisor.

Thorough mastery of decimals in all operations and practical problems.

Speed exercises in addition and multiplication, subtraction and division.

Emphasize fractional equivalents.

Percentage with 50, $33\frac{1}{3}$, 25, 20, $16\frac{2}{3}$, $12\frac{1}{2}$, 10, 5, 75, 40, 80 and $37\frac{1}{2}$ per cent. studied with their fractional equivalents.

U. S. money and thorough mastery of bills.

Liquid and dry measures, avoirdupois weight (problems).

Linear measure with doing problems.

Cubic measure with many doing problems.

SIXTH GRADE.

Factoring.

Simple personal accounts; bills.

Thorough mastery of fractions with problems.

Thorough mastery of decimals with problems.

Thorough mastery of denominate numbers. Connect with mechanical drawing and shop work.

Speed drills in review of fundamental operations; comparison of values in denominate numbers.

Practical measurements; painting; plastering; triangles; volumes and contents with tables and data; lumber measure with real problems. (Check up bills in Manual Training Dept.)

Percentage emphasizing aliquot fractional equivalents where practical; gain and loss; commission; commercial discount.

Interest (6 per cent. method); bank accounts.

Emphasize complete statement of problems before solution. Emphasize doing problems.

Seventh Grade.

The work of the seventh and eighth grades is virtually an enlargement upon the process of previous grades with emphasis upon practical and enlarged application by means of problems of the home, the shop and business of daily life.

Complete review of fundamental processes, factors, divisors, multiples, cancellation, fractions, easy complex fractions, com-

Denominate numbers. Special study of practical measurements, with study of circle, and enlarged practical applications—roofing, fencing, etc.

The equation statement with x for unknown term.

Percentage, gain and loss, commission, insurance, commercial discount, commercial paper, taxes, duties.

analysis with complete statements and short solutions; decimals and business applications; decimal, fractional and per cent. equivalents; bills and simple accounts.

Eighth Grade.

parisons of values including whole with fractional numbers;

Interest.

Saving accounts and compound interest.

Negotiable papers.

Banking.

Exchange.
Stocks and bonds.
Ratio and proportion.
Longitude and time.
Powers and square root.
Mensuration.
Laboratory mathematics, including agricultural problems.
Metric system data.
As complete review of arithmetic as possible.

ARDMORE, PA.

THE BRAVEST BATTLE THAT EVER WAS FOUGHT.

The bravest battle that ever was fought;
Shall I tell you where and when?
On the maps of the world you will find it not;
'Twas fought by the mothers of men.

Nay, not with cannon or battle shot,
With sword or nobler pen;
Nay, not with eloquent word or thought,
From mouths of wonderful men.

But deep in a walled-up woman's heart—
Of woman that would not yield,
But bravely, silently bore her part—
Lo! there is that battlefield.

No marshalling troop, no bivouac song;
No banner to gleam and wave;
But O! these battles they last so long—
From babyhood to the grave!

Yet, faithful still as a bridge of stars,
She fights in her walled-up town—
Fights on and on in the endless wars,
Then silent, unseen—goes down.

O, ye with banner and battle shot,
And soldiers to shout and praise,
I tell you the kingliest victories fought
Were fought in these silent ways.

O, spotless woman in world of shame!
With splendid and silent scorn,
Go back to God as white as you came,
The kingliest warrior born.

—Joaquin Miller.

THE RUSSIAN PEASANT METHOD OF MULTIPLICATION.

By Joseph Bowden.

The Russian Peasant Method of Multiplication is said to be in common use in the villages of Russia, where it takes the place of the method which is in general use elsewhere.*

To be able to multiply by this method it is necessary to know how to add, how to double a number, and how to divide by two, obtaining the exact or lower approximate quotient.† Thus only a small part of the multiplication table need be known. The advantage of the method for persons who find this table difficult to remember is therefore obvious.

The method may be stated in the following rule:

The Russian Peasant Rule (Rule 1).—*Having given the positive integers a and b, to multiply a by b write down* $a \times b$; *under a write the exact or lower quotient obtained by dividing it by 2; under this quotient write the quotient obtained by dividing it by 2, and so on, until you obtain the quotient 1.*

Under b write its double, under this double its double, and so on, until you have as many numbers in the second column as in the first.

Next add the numbers in the second column which correspond to odd numbers in the first.

The result is the product of a and b.

The rule may be illustrated by the following examples:

* I am indebted for my knowledge of this method to Mr. L. Leland Locke, of the Brooklyn Training School for Teachers, and to Prof. David Eugene Smith, of Columbia University. Prof. Smith obtained the method, in a form slightly different from that in which I have presented it, in a clipping from a German newspaper, which quoted the method from the French journal *Cosmos*.

† See Bowden's "Elements of the Theory of Integers," § 513, The Macmillan Co., publishers, New York.

45 ×	24	42 ×	b—Omit
22	48—Omit	21	$2b$
11	96	10	$4b$—Omit
5	192	5	$8b$
2	384—Omit	2	$16b$—Omit
1	768	1	$32b$
	1080		$42b$

That this method is correct may be proved by means of the following theorem:

THE RADIX THEOREM.—*If a is any integer and t an integer numerically greater than one, if we divide a by t, obtaining the quotient q_0 and remainder r_0, then divide q_0 by t, obtaining the quotient q_1 and remainder r_1, then q_1 by t, obtaining the quotient q_2 and remainder r_2, and so on, after a time a number will be reached in the series $a, q_0, q_1, q_2, \cdots$, which is numerically less than t. Dividing it by t, the quotient zero may be taken.*

After the quotient zero is obtained all future quotients will be zero. If $a=0$, the first quotient will be zero. If $a-=0$, no quotient need be zero. But if the quotient zero is obtained (through necessity or choice, according as $a=0$ or $a-=0$), if we stop the process of division at this point, the series of remainders has the following properties:

If $a=0$, there is only one remainder, which equals zero.

If $a-=0$, an infinite number of sets of remainders can be found, the remainders are all numerically less than t, and the last remainder, r_n, is not zero.

Whether $a=0$ or $a-=0$, any set of remainders $r_0, r_1, r_2, \cdots, r_n$ has the property that

$$a=r_0+r_1t+r_2t^2+\cdots+r_nt^n.$$

*If a is positive, $t>1$, and when the division is inexact the lower quotient is taken, the quotients are all positive except the last, the remainders are all positive or zero, the last being positive, they are all less than t, and there is only one set of remainders.**

For example, if $a=94$ and $t=4$, we have the four quotients 23, 5, 1, 0 and corresponding remainders 2, 3, 1, 1.

* See Bowden, "Theory of Integers," §§ 515, 516, 519, 526, 531, 607; Chrystal, "Text-Book of Algebra," Part I., p. 167, Adam and Charles Black, publishers, London.

Hence $94=2+3\times4+1\times4^2+1\times4^3$.

If $a=42$ and $t=2$, we have the six quotients 21, 10, 5, 2, 1, 0 and remainders 0, 1, 0, 1, 0, 1.

Hence $42=0+1\times 2+0\times 2^2+1\times 2^3+0\times 2^4+1\times 2^5$
$=2+2^3+2^5$.

Therefore $42b=(2+2^3+2^5)b=2b+8b+32b$.

This example proves the Russian Peasant Rule for any value of b when a is 42.

To prove the rule for all values of a and b suppose that a and b are positive and that we wish to multiply a by b. Let t be 2 and when the division is inexact choose always the lower quotient. Then we have the three series of numbers given in the three columns below:

a	r_0	b
q_0	r_1	$2b$
q_1	r_2	2^2b
q_2	r_3	2^3b
.	.	.
.	.	.
.	.	.
q_{n-1}	r_n	2^nb,

the last column containing the number b and the numbers obtained from it by successive doubling.

Since the numbers $r_0, r_1, r_2, \cdots, r_n$ are all positive or zero and less than t, and t is 2, these numbers are all either 0 or 1. Moreover, if any number in the first column is even, the division is exact and the corresponding remainder, which is the corresponding number in the second column, is 0. If any number in the first column is odd, the division is inexact and the corresponding remainder, that is, the corresponding number in the second column, is 1. Moreover, since the last remainder, r_n, is not zero, it is equal to 1.

Now we have, by the Radix Theorem,

$$a=r_0+r_1 2+r_2 2^2+r_3 2^3+\cdots+r_n 2^n.$$

Hence

$$ab=r_0 b+r_1(2b)+r_2(2^2b)+r_3(2^3b)+\cdots+r_n(2^nb).$$

The r's are either 0 or 1. If any r is 0, the corresponding term can be crossed out. If an r is 1, it may be omitted as a factor from the corresponding term.

Thus ab is the sum of those of the terms in our last column,

$$b, 2b, 2^2b, 2^3b, \cdots, 2^nb,$$

which correspond to the value $r = 1$ in the second column, or to odd numbers in our first column.

Our rule is therefore proved to give the result asserted of it.

COROLLARY TO THE RADIX THEOREM.—*Every positive integer can be written as a sum of powers of 2 (including the zeroth power of 2, which is 1).*

E. g., $14 = 2 + 2^2 + 2^3$

$47 = 1 + 2 + 2^2 + 2^3 + 2^5.$

Some of my readers will probably be interested in the following two rules, which are variations upon the Russian Peasant Rule:

RULE 2.—*To multiply a by b write down in a vertical column the number a and the successive quotients, exact, lower, or upper, obtained by dividing it and the series of quotients by 2, until you obtain the quotient 0.*

In another vertical column, starting on a line with the first quotient, write down the number b and the numbers obtained from it by successive doubling, until you have a number in the second column for each quotient in the first. Next add the numbers in the second column which correspond to lower quotients and those which correspond to upper quotients. Subtract the latter sum from the former. The result is $a \times b$.

E. g.,

	27		60	15
Upper	14	15	120	240
Exact	7	30	480	255
Lower	3	60	660	
Lower	1	120	255	
Upper	1	240	$27 \times 15 = 405$	Ans.
Lower	0	480		

RULE 3.—*The same as Rule 2 with the following changes:*

Divide by 3 instead of by 2 and always choose the quotient which gives the remainder 0, 1, or −1. Triple instead of double.

E. g.,	48		63	189
Exact	16	21	1701	567
Lower	5	63	1764	756
Upper	2	189	756	
Upper	1	567	$48 \times 21 = 1008$ Ans.	
Lower	0	1701		

Innumerable other rules may be derived from the radix theorem by giving t various values, positive and negative.

These other rules, however, are not so simple in statement as the three given, except the rule obtained by taking t equal to 10. This rule is practically the same as the ordinary rule for multiplication.

EXAMPLES.

1. Express as sums of powers of 2 the numbers 1, 2, 3, $\cdots$, 16.
2. Express as above the numbers 87, 96, 436, 783.
3. Multiply 96 by 87. Check by multiplying 87 by 96.
4. Multiply 783 by 436. Check by congruences to the modulus 5. Check also by modulus 6 and by modulus 4. (*If* $a \equiv b$ *and* $c \equiv d$, *then* $ac \equiv bd$.*)
5. Multiply 3,689 by 2,728. Check by casting out the nines.†
6. Multiply 359 by 422. Check by casting out the threes.
7. Multiply 8,979 by 7,639 and check by casting out the elevens.

$$[a \equiv (r_0 + r_2 + r_4 + \cdots) - (r_1 + r_3 + r_5 + \cdots).‡]$$

BROOKLYN, NEW YORK.

* See Bowden, "Theory of Integers," §§ 731, 753, 760; Chrystal, "Text-Book of Algebra," Part II., p. 500.

† See Chrystal, Part I., p. 175.

‡ See Chrystal, Part I., p. 178, ex. 27.

WHAT MINIMUM OF PRACTICAL APPLICATIONS SHOULD BE INCLUDED IN A COURSE IN ARITHMETIC.

By C. R. Dooley.

The Student.

Whatever may be our definition of education it surely must in some way help the student toward a fuller and happier life—a life more abundant. The oldest thing in the world is life and therefore it is the one thing that holds our interest through all our years. Life is the application of all that we know and are and feel and whether we admit it or not any sort of formal education is the incidental thing and all classes of students make it so in spite of pedagogic pressure—and it is so. Practical applications, therefore, are reasonable, first of all because of the vital interest they afford. Any subject is most efficiently taught under those conditions which provide the maximum interest on the part of the student, and this is always obtained through some form of actual contact or association with the real things of everyday life. We all know that arithmetic, algebra, geometry, etc., are means and not ends, but we do not teach them that way.

It requires but a few practical problems to show the student the real use of a mathematical principle—to show that by a simple calculation the task becomes easier and time and effort are saved. The student gets a broader view of his work, or even of his play, sees all sides of his problem at once, the possibilities, the limitations and its relation to other problems. He feels he is working effectively. He is enthused over the great assistance his brain is able to render his hands. He adds to his interest inspiration and develops ideals of efficient service.

Formal education must accomplish two things: *First,* it must lay a foundation upon which a student may build as high as he is able without any backward step. *Second,* it must enable a student to step out into life at any stage of his schooling where either circumstances or his natural limitations call him, fully

able to use his mind and body to his best ability. And we might add a third, viz., all students should leave the school room, however high they have climbed, inspired to continue their search for knowledge, but knowing that the world will always ask of them "What can you do?" and never "What do you know?"

A young man or a young woman completes the prescribed college course and we call it commencement, but the graduate does not get the idea till ten years later. The boy or girl who leaves school at the seventh grade, however, never hears anything about this commencement, except as a thing he has missed and glad of it. Neither his teacher nor his text have fitted him for a useful life, even though he far outnumbers those who get a sheepskin.

Just how far down into the grades the things of our daily life should penetrate, will probably always be a question and the answers will vary with each locality, but we are generally agreed that at present the change from school life to real life, at all stages, is too abrupt. In the subject of mechanics a square butt joint can never be as strong as a tapered splice.

The Teacher.

The formal teaching of any subject is a hard task for the teacher but there are at least two still more difficult. *First,* to so inspire all students that the lessons unconsciously become vital living parts of each life and not mere dead pages of a text book to be memorized. This may be a matter of proper personality which all teachers do not have. They should have it and those who do not—well, they are in the wrong job and for the sake of the children should be removed at once. Many a first-class office girl or stock clerk is struggling to make a fourth-class teacher. *Second,* to so know life and humanity as to accurately determine the capabilities of each student and then direct him along the line for which his natural qualifications best suit him.

There has been far too much of a tendency among the common and secondary schools, toward influencing any and all students to take higher educations. The result is that today there are many students struggling their way through college who

ought never to have gone to college and who will never be able to make anything like efficient use of their training. Of course this training in one sense has not done them any harm and yet the time would have been more profitably spent in some other kind of training, either systematic or through experience, which would have assisted them toward a greater success, but society says "You must have a college diploma to be really respectable." I speak with some knowledge of this subject, since the company with which I am connected offers practical training each year to some two hundred graduates of technical schools, many of whom should be farmers, store clerks or policemen. And in these positions they would be successful and therefore really more respected.

It will require a new kind of teacher to recommend certain children of wealthy families to leave the formal school and entirely change their process of education and it will require a new kind of school authority to provide many kinds of training, instead of one.

The teachers of the future will be most carefully selected and trained in the same manner and according to the same standards which they, in turn, will apply to the students. They will further be required to keep in touch with practical life through regular commercial employment, at proper intervals.

W. E. & M. Company—Educational Work.

Industries are training their young men because conditions have forced them to it and not because they know better how to do it. Industries may always have to do a certain amount of detail adaptation but today they are unquestionably supplying a line of elementary instruction to their apprentices, which the public schools should do.

The educational work of the W. E. & M. Company at Pittsburgh is divided into three parts: *First, The Graduate Student Course,* open to a selected number of graduates of technical colleges. This course is two years long. Each student is required to attend class work, on company time, one or more half days per week. At present there are some 300 students on the course. *Second, The Trades Apprentice Course,* open to a selected number of boys from 16 to 21 years of age. This course

is four years long. Each student is required to attend class work, on company time, four hours per week, at which he is given systematic instruction in "shop problems," "mechanical drawing" and "English." There are now some 200 students on the course. *Third, The Casino Technical Night School,* open to any man, woman or child without exception. The course varies in length, according to the ability of the student, up to 8 years. A small tuition fee is charged which covers about half the operating expense. There are now about four hundred students in attendance and 40 members of the faculty.

All of this is costing a great deal of money not as an experiment but as an investment which is yielding good results.

Shop Problems as Taught in the Trades Apprentice Course and in the Casino School.

As a rule boys come to us thoroughly disgusted with school. They want to work. We give them work, but they also have to go to school 4 hours each week and they soon get to like it—some so much so that they return to public school with a new vision and appreciation.

No text book is used. The problems are all made up by an instructor in charge of the work. A few characteristics of these problems are notable. *First,* the language and names of things used in stating the problems are those met with in daily life about the factory. *Second,* there are no abstract problems, such as $\frac{3 \times 8}{\frac{1}{4}}$. Each problem is a definite practical statement closely related to daily experience. *Third,* no hard and fast divisions of "arithmetic" such as "decimals," "fractions," etc., are held up before the boys. *Fourth,* no formal rules or methods are set down to be committed. *Fifth,* a problem is more than a mere problem—it is a statement of real conditions and often more data is given than is needed, requiring proper selection and statements. While little in advance of the arithmetic is taught yet the formal word "arithmetic" is not used. The boys are graded into several classes according to the classification of the ordinary arithmetic in a very general way, but at all times they are studying "shop problems" and not "decimals" or "percentage." The problems are to be solved by

analysis and the applications of general principles, without regard to whether these principles belong to "Proportion" or to "Mensuration." Each principle is explained only after a need for it has arisen.

We have found boys whose previous schooling had been so formal that they could not do long division without placing the dividend, divisor and quotient in certain definite relative places. A student should so comprehend such an operation that he could put the dividend on the black-board; the divisor on a scratch pad and write his answer on the floor; and he may do all this in a neat and orderly manner. Neatness and order do not mean conformity to one definite form laid down by the law.

Rules are not given with the command, "Learn these for they will be useful later on." But the boys are put up against problems of real shop experience and then by the guidance of the instructor, the proper method is explained and the student assisted in formulating his own expression of such rules as are necessary. I gave this problem to a class of fifteen apprentices: $\frac{3 \times 8}{\frac{1}{3}}$; and they solved it readily. Half an hour later the following was given to the same class: "How many bolts each 4" long can be made from three brass rods ½ inch in diameter and 8 feet long?" Not one saw the similarity between this problem and the previous one. Some asked what to do with the ½ inch diameter and one thought bolts were made of iron, and the brass in this problem bothered him. After a little suggestive discussion, about half the class fully comprehended the problem and its solution.

Another example will illustrate the practical application of fundamental principles. This problem was given one of the *advanced* classes and was readily comprehended and solved.

"A casting weighing 4,000 lbs. is to be raised by means of a crow-bar. The distance from the point of the bar under the casting to the pry-block is four inches; the distance from the pry-block to the point where the bar would be grasped is four feet." The question asked was not how many pounds weight will balance the load, but how many men will be required to raise the casting.

Immediately some one asked how much a man weighs and the

reply was, "Use your judgment, would you send one man or ten to do the job?"

Then the problem was carried further. "How high will the casting be raised?" They soon decided that it could be raised about one inch. Then they were asked, "How many men will be required to raise it ten feet, and the answers came thick and fast all the way up to 120 times as many men. By-and-by one boy said, "two men—just keep blocking up the casting," and another said, "then you'll need another man, one to fix the blocks"; and so on leading the boys to think out rational solutions to everyday problems by the application of the fundamentals of arithmetic. In the case just cited they were dealing with proportion but they did not know it as a formal, dead and difficult part of the arithmetic to get through with on examination day.

The majority of our boys have left the public school at the seventh grade and have been out of school from one to two years. Out of 100 who took a classification examination, 50 could not add decimals or complex fractions and but 25 could *at all* handle problems in simple percentage. The great majority had absolutely no conception of the physical meaning of problems in areas and volumes. I have asked many new boys to find the area of the top of my desk $4\frac{1}{2}$ feet by $5\frac{1}{2}$ feet, and received the answer $4\frac{1}{2} + 5\frac{1}{2} = 10$ or "I have forgotten the rule."

These cases are not exceptional; they represent at least 50 per cent. of the boys who come to us. Nor are these boys stupid and dull. On the whole they are wide awake and of good appearance. They take readily to leading suggestions and are making wonderful progress in their application of fundamental principles to the practical problems of their daily lives. Interest runs high, for they find their shop work is easier and more intelligible.

As far as possible the boys are never required to take anything for granted because its derivation is based on "higher mathematics." *For example:* The value of pi is proved quite to the satisfaction of all by providing a dozen or more discs of different diameters. Each boy measures the diameter and circumferences of these discs and works out the ratio in enough cases to satisfy him. The specific weight of different metals is

also worked out by measuring and weighing numerous samples in the class room. This creates interest and fixes values in the memory. The apprentice knows that a cubic foot of cast iron weighs 450 pounds because he has calculated it a dozen times with as many different pieces and has checked his results with the scales.

We give a boy a pulley, a piece of rope, a weight and a spring balance and tell him to find out all he can. What force is required to lift the weight? Does this force change with pulleys of different diameters? Does it change with ropes of different sizes? Where have you seen a similar use of a pulley in the shop? Can this apparatus be arranged so that a force of 5 pounds will lift a weight of 10 pounds?

After he is through with his test he has the laws of the single pulley fixed in his memory in his own language. This is carried on through hundreds of combinations with pulleys, levers, gear wheels, weights and measures, screws, force combinations, etc. All this is mechanics and is interwoven with arithmetic, algebra, geometry and trigonometry, but these are absolutely incidental to the physical appreciation and solution.

A few months ago we established a series of talks by the instructors on shop topics such as, "Machine Tool Parts," "Small Tools," "Standard Bolts, Nuts and Washers," "Castings," "Brass," "Iron," "Steel," etc. At the end of a four months' term, an examination was held with the result that those talks which had been illustrated by objects had been universally remembered and the others largely forgotten.

Practical Problems for the Public School.

Select the things in which each class of children is interested and build the problems about them.

An associate of mine recently told me that his boy of ten was much distressed because he was unable to teach the housemaid how to add. She could not seem to understand his explanation that: "Addition was the process of taking . . ., etc." When asked if she brought home the correct change with the grocery order, he replied, "Oh yes, she can add it all right, but she can't understand the rule."

Grocery store calculations are full of additions, subtraction, multiplication, division, percentage and fractions.

If a 4-lb. roast cost 25c. a lb., and afterward the bone weighed 12 oz., what did the actual meat cost? What is the percentage of waste?

What is the diameter of your bicycle wheel? How many revolutions does it make to the mile? How many revolutions of the pedals to the mile? Is there any ratio between the proper of teeth on the two sprockets and the revolutions per mile of each?

The small boys' wagon has wheels of 8 and 10 inches diameter. How many revolutions does each of the four wheels make if the wagon is pulled once around a circle of 10 feet diameter?

"What are the percentages of each member of your baseball team?"

You deposit $1.00 per month in a savings bank, paying 4 per cent. per year, interest being credited to your account every six months. How does your account stand at the end of 17 months?

Material for a skirt costs 75c. a yard. Seven yards (or 2½ yds. nowadays) are required; thread and trimming cost 50c. What is the cost of the skirt? How long would it take you to make it? Can you afford to learn to make such garments if the ready-made price is $15.00; and what could you save per year by so doing?

Determine the height of the school building without actually measuring it.

Find the dimensions of each board used to make a box which will just contain a can of carbon oil 41″ high × 17″ diameter. If it weighs 100 lbs. and is to be shipped from Chicago to St. Louis, which will be the cheaper, by express or freight, and how much? Which will be the quicker and how much?

How many days till the Fourth of July, and each day what percentage is the time reduced from the preceding day? Show the result by a curve.

Each student should be required to collect his own data from his own experience and investigations. To carry out such a scheme, we should have 100,000 problems, classified as to subject matter (not as percentage, proportion, etc.), under a hundred different heads and then each group arranged according to degree of difficulty. Then, day by day, each student should get that kind of problems that will catch his greatest interest. This

means individual instruction, but we've been too long already with aiming at the average of a class of 40 boys and girls, no two of whom are anywhere near alike in any way.

Our present system is perhaps not bad for the average child and those below, but it is sure death to the natural powers of the bright ones.

Conclusions.

Practical applications in a course in arithmetic will do the following:

1. Create the greatest interest.
2. Lay the best foundation for future mathematics.
3. Give the maximum utility at any time it becomes necessary for a student to leave school.
4. Provide a sure means of sizing up students and directing them toward that life work for which they are best fitted.
5. Inspire the student with ideals of labor and citizenship.
6. Encourage a spirit of eternal inquiry, investigation and observation.
7. Provide a certain knowledge of the things and of the people of the world.
8. Give some appreciation of wholesome economies and a sense of proper relationships and proportions.

The minimum of practical application which should be included in a course in arithmetic is not far from 100 per cent., and the application should precede the principle.

Westinghouse Electric and Mfg. Co.,
East Pittsburgh, Pa.

APPLIED ARITHMETIC.

By Louis L. Park.

A prominent professor of engineering in one of our eastern universities when asked "what becomes of the men who graduate with high honors?" replied "We seldom hear from them after graduation. They are too theoretical; no one wants them."

While we may feel disposed to question the accuracy of this statement or the extent to which it was intended to apply to all branches of university work there has been given by other college men sufficient testimony of the same character to lead us to believe that many of those who stand highest in mathematical subjects during their school courses are least fitted to make use of the knowledge acquired when confronted by the problems of practical work.

We must admit that this word "practical" has been somewhat overworked in late years in connection with proposed reforms in educational methods, but it remains true that the ability to make use of the knowledge gained at school is the final test of the value of one's education. Not the ability to juggle figures and get an answer, but the power to command the service of mathematical principles must be the measure of our success in teaching arithmetic, and the question before us at this time, from our viewpoint, is "What method in arithmetic will bring the subject most fully within the grasp of the student as a means of enabling him to solve problems in everyday affairs?"

As the skilled workman has before him a rack of tools of various shapes and uses, from which he selects, as his work progresses, that instrument which enables him most readily to perform the operation needed, so to the student trained in the use of mathematics, do the different branches of the subject stand at his disposal ready for use when required. It is not enough that he know their names and be able to handle them quickly; if his work is to amount to anything he must know which tool will do the work most readily, and he must be able to use it to advantage. The greater his skill, the less is he con-

scious of the tool in his hand, and the closer is his thought fixed upon the work before him.

Naturally much of this skill must be attained after leaving school and the question which confronts the school is, "how much should we be expected to train the student in the use of the mathematical tools for practical purposes?" Unfortunately with most young people leaving our schools, several years elapse before the arithmetic handled in school is brought into action in business problems.

Unlike its two sister R's, arithmetic does not usually enter as fully into the daily life of the student, either during the school period or immediately after.

Reading is one of the accomplishments whose utility is never questioned by the youthful student. He needs no studies in "applied" reading for he finds ample use for his accomplishments as fast as acquired.

So also is his attitude likely to be toward the art of writing, for he is confronted by so many incentives to write that its practical value is proven almost daily.

Beyond the very elementary steps in arithmetic we believe that this subject does not appeal to students as being of the same daily necessity, as are the other subjects, and it therefore is not absorbed as fully as it might be.

Unless a man live in the wilderness he cannot forget to read so long as his eyesight holds out, and while he may become a poor writer and a bad speller, he can usually make himself understood, but if he be very long out of school and have no demand placed upon him for work in arithmetic he soon becomes rusty and loses most of his ability to handle problems requiring much figuring.

So largely does the knowledge of arithmetic depend upon memory that with most people, when it is lost it is gone forever unless again acquired through study; still further, the loss of the principles or of some details of arithmetic through lack of use is made worse by the inability of the individual to bring back these details through reasoning. Nearly fifty per cent. of the boys applying to us for positions as apprentices cannot recall the number of cubic inches to the cubic foot and some of those who do not know make a wild guess at it hoping that luck is with them. The sad part of it is that they do not

know how to get this information without referring to a text book. Whether this lack be due to wrong methods in teaching or to the lack of interest on the part of the student while at school we may not know, but it would seem that it were possible to enable students to think their way into arithmetic after leaving school if some effort had been made to lead them along by this method while in school.

Probably all text books in arithmetic for primary grades abound with concrete problems which are both interesting and helpful to the student and usually the concrete is made to introduce new steps and precedes the abstract. In most text books for grammar grades, however, the various steps are first dealt with in the abstract and then followed by concrete problems. Such problems are in most cases sufficiently ample to illustrate the scope of the operation involved and its application to practical work, provided the student understands the value of the application, for in many cases the problem is put in such form that the student simply recognizes in it the process of the abstract dressed in other garb and he falls back upon the abstract for assistance. If, then, the abstract be lost, there is nothing left to help with the concrete.

Personally, I believe that the principle followed in the elementary text books by which the studies pass from the concrete to the abstract is worthy of being followed throughout the entire course. Some striking illustration of a new principle will often help to fix it in the student's mind and serve as a starting point for calling back the lost principle. Mr. Howard Renshaw, of the continuation school in Cincinnati, tells of introducing work in decimals by getting the boys to work up a simple shop payroll. We doubt if these boys will soon forget that payroll, and it is likely to prove of help to them in keeping in their memories the principle involved.

To most students concrete problems are far more interesting than studies in the abstract and interest in the abstract can be more readily obtained if it is found to be necessary in solving some practical problem. A teacher in one of our evening classes for shop boys wishing to give them instruction in fractions (in which study they were all deficient) held up a casting and asked them to estimate the weight. The guesses ranged from 15 to 30 lbs. The class then undertook to calculate the

weight and as the dimensions of the casting were found to come in fractions of inches the result was a lesson in fractions. Several evenings were spent in work of this sort and the interest was such as could not possibly have been obtained had the instructor said "boys, we will now spend the evening in studying fractions," and then giving them the usual exercise in addition and multiplication of such quantities.

It has been our experience that apprentices make much better progress in arithmetic when studies are made to lead from the concrete to the abstract, from the illustration to the principle, than was found by the old method, which was tried at the start.

In the selecting of practical problems it is, of course, desirable that, at the start at least, they be as closely related as possible to the student's interests. The more realistic the problems the more his interest in them. Problems originated by the student himself should be encouraged, as they help to strengthen his ability to apply the principles taught. Still further, the giving of problems in which all details are not stated and in which he must get part of the data from other sources are of the greatest value to him.

Book problems usually give all of the required information necessary to obtain the answer but if the student be asked to go to another room to get measurements first hand, or be asked to measure up articles at home, it is likely that he will find upon his return that he has missed something or that the figures he did get will not tally. Few boys from our schools can be sent into the shop to make a sketch of some simple piece and return with all the data needed, and the apparent cause of it is that he has had so little occasion to do work of that sort that he has not learned the importance of being sure that his work is complete and correct before handing in his results. It may truthfully be said that such work is not properly part of the study of arithmetic, but it has its bearing upon the relating of arithmetic to real life.

This suggests that which, perhaps, is the greatest fault of the graduates of our schools, namely, the inability of so large a proportion of students to solve problems which call for original thought, problems confronted in actual business life, problems not bounded by definite mathematical limits and not classified under any special group of principles.

Many of the so-called practical problems used in text books are mere repetitions of each other with different names used for the setting of the problem. The student has no interest in the articles referred to, be they bricks, or rails, or beans, or hours, but he recognizes in them the familiar form of the abstract process and works the problems on that basis. The style of many such problems may rightly be termed "a subterfuge," a "delusion," when considered as concrete problems, for they are merely a play on words and give no idea of the solution of problems in real life.

The writer not long ago visited an 8A grammar grade class in arithmetic, and found it engaged in a study of ratio and proportion. The students were sent to the blackboards and given problems calling for the use of the subject being studied and several passed successfully through the computation of how long it would take 8 men working 8 hours a day to mow a field of oats, if 6 men working 10 hours per day did it in four days, and so on. All went smoothly until one small boy struck a problem in leverage and he succeeded in getting a weight of 150 lbs. on a 78″ arm to balance a weight of 15⅓ lbs. on an 8″ arm. Both the teacher and the class passed the solution as correct, but the visitor was curious to know how such an impossible feat was accomplished and plied the boy with questions about the lever. It transpired that the class knew nothing of the principle of leverage although it was stated in the text book, nor had they any idea of what the lever was used for. The teacher made things still worse by explaining that these problems were given, not as a study in leverage, but simply as practice in ratio and proportion.

The visitor was then asked to give the class a problem in proportion and he, being accustomed to shop ways, sketched on the board a plain tapered bolt, gave the diameter at the small end, the taper per foot and the total length, asking them to figure the diameter at the large end. In a few minutes one boy had an answer, and by request showed it to the visitor who was startled to read the figures "510." One girl hit the proper trail and obtained the correct answer, but so far as the visitor could determine after ten minutes work the rest of the class were far adrift and never got within sight of land. Had the visitor stated the problem thus "If a bolt tapers 1/16″ in 12″, how

much will it taper in 27″?" the whole class would have had an answer in two minutes, but, robbed of its familiar form the problem was hopeless. Most of the members of that class had passed regents, and were ready to enter high school, or, many of them, to go to work. May we not be justified in questioning the ability of these students to make practical use of arithmetic.

The thing which destines most men to success or failure in business is their ability to surmount difficulties, to solve practical business problems. Undoubtedly it is this which caused the remark quoted in the opening paragraph of this paper and it is this which leads to the drawing of a line of demarkation between the men of theories and the practical men.

We have no intention of claiming that the cause or cure of this trouble lies within the field of mathematical study, but in the common school curriculum there is no subject which offers better opportunity to the handling of practical problems than does arithmetic, and it is for the purpose of encouraging the full use of the opportunities of the subject in this direction that the question is here raised.

It is only as the problem work leads to the use of arithmetic as a tool for accomplishing results and the student recognizes that arithmetic exists because problems demand it and not that problems are needed because arithmetic requires them, that he is headed in the right direction. The ability to properly analyze a problem, determine the steps by which we may work from the data in hand to the information required in the answer is by far the most important part of a study in arithmetic and it is upon this phase of the subject that we believe the emphasis now needs to be placed and for this purpose should the so-called "practical problems" be made more largely to serve.

May we say then, in answer to the question which we are discussing this morning that the minimum of practical application in a course of arithmetic should be sufficient to provide, first, the securing of attention and interest; second, some idea for the reason for each process studied; third, opportunity for the analyzing and reasoning out of problems, and, fourth, a grasp of arithmetic sufficient to make it a tool for obtaining results.

There can, of course, be no general limits to the term "suffi-

cient" and it must vary with the individual needs and the time available. We are aware that this question of time is a serious one and one which prevents teachers from doing what they would like to do along the lines suggested. Our manual training schools have done much toward making more practical the use of arithmetic and have increased somewhat the time devoted to the subject. As an indication of the more practical character of the work done by some of our manual training schools it might be said that the writer gave to a seventh grade class in one of these schools the problem of the tapered bolt referred to previously and 10 out of the 17 members obtained the correct answer in a few minutes, without any help from the teacher. We believe however, that with the proper presentation of the subject the time available in the ordinary grades can be used to better advantage than is being done by some classes at the present time. With proper appreciation of the subject in general, the student will take greater interest in the drill which is necessary to obtain quickness and accuracy. One can hardly overestimate the value of training on these points, particularly the latter, for the habit of carelessness is one of the greatest handicaps with which a young man or young woman starts a business career. Promotion comes to those who are dependable and whose work can be relied upon as being correct, and without the habit of accuracy being formed early in life a long and painful struggle is usually required to accomplish the needed change.

The mental discipline represented by these desirable traits will, of course, be accomplished through work in the abstract as well as with concrete examples, the former for speed of operation and the latter for quickness in choosing the method to be employed; correctness of result being equally important in both.

In deciding upon the amount of practice desirable in handling concrete problems in arithmetic, we do well to shape our course to meet the needs of the average student, providing extra work for those who naturally run ahead, remembering that the great majority of those who pass through the grammar grades will go directly into business, leaving only a small percentage to finish high school. Many of the former will never be called upon to use more than the most elementary of the principles of

arithmetic but to a fair proportion will come the opportunity to rise to places of greater usefulness through the ability to think beyond the limits of mere necessity and the mental discipline of a well-planned and a well-taught course in arithmetic will be one of the means by which they rise.

AMERICAN LOCOMOTIVE CO.,
SCHENECTADY, N. Y.

A PSALM OF THE GOOD TEACHER.

The Lord is my teacher:
I shall not lose the way to wisdom.

He leadeth me in the lowly path of learning,
He prepareth a lesson for me every day:

He findeth the clear fountains of instruction,
Little by little he showeth me the beauty of the truth.

The world is a great book that he hath written,
He turneth the leaves for me slowly:

They are all inscribed with images and letters,
His face poureth light on the pictures and the words.

Then am I glad when I perceive his meaning,
He taketh me by the hand to the hill-top of vision:

In the valley also he walketh beside me,
And in the dark places he whispereth to my heart.

Yea, though my lesson he heard it is not hopeless,
For the Lord is very patient with his slow scholar;

He will wait awhile for my weakness,
He will help me to read the truth through tears.

Surely thou wilt enlighten me daily by joy and by sorrow:
And lead me at last, O Lord, to the perfect knowledge of thee.

—*Henry Van Dyke.*

WHAT MINIMUM OF PRACTICAL APPLICATIONS SHOULD BE INCLUDED IN A COURSE IN ARITHMETIC? WHAT ADDITIONAL APPLICATIONS SHOULD BE INCLUDED FOR PURPOSES OF MENTAL DISCIPLINE?

By William Reed.

The subject which you have assigned to this paper is one of extreme importance; much more so than is generally supposed. The development of our foreign trade has brought into sharp comparison the American and foreign born scholar, especially the German, and many business men believe that our own people do not stand the test satisfactorily. As against this I am inclined to think that most men lean toward the idea that the so-called "old time methods" were best because that was the way they were brought up and a natural prejudice exists in their minds in favor of the systems with which they are familiar. There is more or less sentiment in that view so that in considering "the pessimistic utterances of business men" on this general subject, some allowance should be made for bias. If an attempt were made to thoroughly teach business practices in the schools, a competent graduate might still be criticised, by a short-sighted business man, for lack of technical knowledge and for not being able to immediately grasp the intricate details of a particular business or trade. That would be extremely unjust, for no two concerns manage their internal affairs by identical methods. Even in banks, whose business is so carefully defined by law, differing styles of bookkeeping are in active use, so that an experienced clerk, changing from one bank to another, would find many new things to learn as far as system is concerned. But it *is* true that the German youth who comes to this country to engage in banking or other lines of business seems to be better equipped at the outset than the American boy of the same age and station in life. This is unquestionably due to system in schooling, for I doubt if any foreign people possess greater natural advantages than do we. In discussing a

question of this kind, too, everyone is inclined to consider only his own point of view, and it may be that absence from school for so many years has developed rigid ideas in my own case and I may be mistaken, but from what I have been able to observe I am convinced that our modern public schools attempt too many things at the expense of thoroughness in the essentials.

To teach every boy and girl drawing and music, biology and snatches of difficult science, without any regard for aptitude, is to the business man, absurd. It might be wise to establish classes in those and similar studies for scholars who naturally lean to them or whose parents express that desire. But there is a well-defined opinion held by many people that these specialties are frequently established not for the sake of the pupil, but to gratify the whims of faddists. If a child has special ability along special lines, it should be developed in a special place or class, but not in the general classes of a public school If all the hours of school study were completely given over to arithmetic, grammar, including reading and writing, modern history, geography and languages, I believe the results to the commercial world would be immensely improved. And even in these studies the predominating feature should be to train the pupil to *think* rather than to cram him full of information. Experience and training make me more sympathetic with your particular branch of teaching than any other, and I should like to see it emphasized still more in the public school curriculum. *That* and modern languages constitute the real need of the boy graduating from public school.

The very young scholar should, of course, be taught to add, subtract, multiply and divide, and in this part of his studies I believe in thorough drilling, so that as he advances into deeper water no time need be wasted on that feature. The young pupil is competent to attain to very great dexterity in the elementary classes, and if he is properly trained the teacher of advanced grades is relieved of a great burden for the attention of the scholar can then be fixed and stay fixed on the explanation of an example and not be diverted by the struggle with the mechanical part of the work. This is such a simple thing that it hardly seems worth mentioning, but I have no doubt that many of our difficulties lie right there. I believe also in thorough

teaching of fractions and decimals, not at first with a particular application, but in an abstract way only. Then, when mastery of the operation of these is obtained, begin to apply. And it is at this point in the child's studies that the skill and experience of the teacher are put to the test. You know very much more about that than I do, and it is unnecessary to dwell on it, but we all know that some possess in greater degree than others the faculty of explanation.

After complete facility in figuring is acquired, I think the reason for every step should be made clear and the pupil made to go through it audibly, so that not only is he taught methods but the habit of reasoning is developed, then if a trifling change is made in the nature of a given problem the consequent change in reasoning would instantly occur to the mind; the idea being, as I have already noted, not to accumulate information but to *teach thinking*. I believe that a good teacher does a great deal of talking. Examples in measurements, costs and profit and loss are essential and I understand are very generally adopted, but let me put in a plea right here to make your examples as near to real conditions as possible. For instance, in giving an example to ascertain the number of yards of carpet at a certain price needed to cover a room of a given size, make your price about what a moderate carpet really would cost and the width of the breadths the standard size. And one other thing. I don't believe in teaching "short cuts" *until* the long way round is thoroughly understood. Only a short time ago I was amazed to find my daughter doing some home work and attempting a short method by a process of cancellation just because she was told to do so. She had no idea how to do it by the process of reasoning. I was taught in school, in figuring interest, to multiply by the days and divide by six to ascertain what six per cent. of an amount would be. I got the decimal point in the right place only because I realized, by a sort of intuition, that it couldn't be so small an amount if put in one place nor so large an amount if put in another. It may surprise you to know that I had been employed in a bank some time before it occurred to me to figure out that thing and explain it to myself, and since then I have had many opportunities to teach others who expressed as much surprise as I experienced. I would teach a child how to render a bill at the same time simply and tactfully,

explaining the processes of business. How economy is practiced by men devoting their whole energies to the production of one thing and the manner of procuring the other necessities of life by purchasing them with the proceeds of the sale of what they produced. Then move on to discounts; showing how goods can be bought cheaper for cash than on time. From this naturally would develop the fact that business as now conducted requires a great deal of money, that it is perfectly proper for men to borrow money to pay for raw material and wages, but that the sooner they are paid for what they produce the shorter the time required to use borrowed money, and the sooner they are out of debt the smaller the expenditure for interest. This will also indicate the advantages in *giving* a cash discount and will lead up to the subject of bank loans and interest and bank discount. You will observe that I am now emphasizing *application* which to my mind is essential at every step after the early period of grinding drill. I wouldn't teach anything without showing an application, and I am certain I couldn't learn without it. Interest and discount should embrace a thorough training in compound interest and discount, for a large percentage of business transactions calls for the exercise of those calculations. As arithmetic is generally looked upon, the divisions which I have mentioned do not seem to be very comprehensive, but I feel certain that the average business man would be completely satisfied if a young man entering his employ had a thorough mastery of them. No reasonable man expects a student fresh from school to take right hold of problems peculiar to his affairs. In fact, I think most men would rather have one with mind perfectly clear of preconceived notions, so long as he could perform with facility a question in mathematics after the local system had been explained. If a boy has been so trained that he can keep his mind fixed on what it is he is endeavoring to ascertain, and shows a fair degree of dexterity in handling figures anyone who possesses the power to make his meaning clear can mould the novice into his ways without difficulty. I think it is almost useless to attempt to teach exact business methods except as to general principles, for no two systems are exactly alike. In other words, the pupil should have a "thorough knowledge of integers and frac-

tions and facility in handling them and above all the power of mind which a proper instruction in the subject is capable of giving." Teach neatness, teach accuracy, teach speed in the subject mentioned, and if you turn out your boys and girls in that condition we will do the rest.

NATIONAL CITY BANK,
NEW YORK CITY.

Whatever I have tried to do in life I have tried with all my heart to do well. Whatever I have devoted myself to, I have devoted myself to completely: in great aims and in small I have always been thoroughly in earnest.—*David Copperfield.*

EDUCATIONAL VALUES OF GEOMETRY.

By F. F. Decker.

It is doubtless quite unnecessary to point out to an audience of members of the Association of Teachers of Mathematics in the Middle States and Maryland the nature of mathematics or in particular of Euclidean geometry; yet because of its importance as a foundation for an intelligent discussion of methods of teaching geometry we shall refer briefly to it.

Mathematics is, not to attempt too great a refinement, the science of implications; and its work therefore consists in formulating postulates and tracing the inevitable conclusions. Euclidean geometry, the first geometry for the race and for the student, is characterized, in addition to its indefinables and the laws of deduction which it postulates, by its own postulates. The work of the student is to comprehend them and to make various chains of deductions. To perform this work is to study geometry.

Geometry as a science has then no connection with the phenomena about us—is not a natural science. But to assist the student to master the subject the teacher may find it necessary, yes, will many times find it necessary, to refer to the child's sense perception. In forming the concept of Euclidean space one will probably make a comparison with physical space. Thus if free mobility is to be postulated, the difficulty of moving a physical body without changing its size on account of variation of pressure may be illustrated by the case of a balloon sinking in the ocean. Again the difficulty of moving a physical body to any position on account of the impenetrability of physical matter may be pointed out. The former illustration (concerning change in size due to pressure) will suggest to the teacher that a geometry may be constructed containing the postulate that the size of the figure decreases in proportion to the distance it is moved. The successful teacher's mental storehouse will contain many such comparisons to be used when needed, but they will not be considered as geometry.

Again all teachers will have the pupil draw figures, but will

consider what seems to happen on the figure as drawn as a matter to be investigated rather than as a deduction. For example in the usual proof of the proposition that every point in a plane equi-distant from two points A and B is on the per-

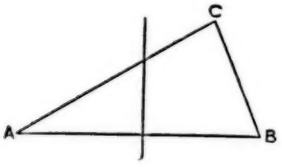

FIG. 1.

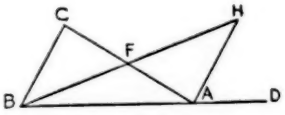

FIG. 2.

pendicular bisector of the line segement joining them by pointing out the inconsistency in assuming that such a point C may not be, one of the lines CA or CB in the figure as drawn seems to cross the perpendicular bisector of AB.

As another instance, in the proof of the theorem that an exterior angle of a triangle ABC is greater than either opposite interior angle, says B, the line segment drawn from the vertex A to the end of the line segment BH drawn to the middle of the opposite side and then produced its own length—the line segment AH—seems to be within the exterior angle so as to make the angle HAF which is the equal of ABC a part of DAC.

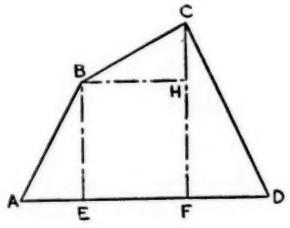

FIG. 3.

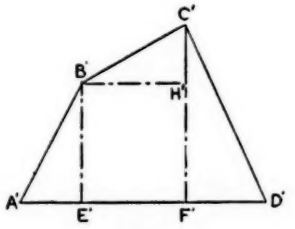

FIG. 4.

We shall give as a third instance what may appear to be a proof of the theorem that if two quadrilaterals have three sides of the one equal to three sides of another, each to each, and the two corresponding angles adjacent to the fourth sides equal, each to each, the quadrilaterals themselves are equal.

Let the given quadrilaterals $ABCD$ and $A'B'C'D'$ have $AB = A'B'$, $BC = B'C'$, $CD = C'D'$, $\angle DAB = \angle D'A'B'$ and $\angle CDA$

$= \angle C'D'A'$. Draw BE and $CF \perp$ to AD, $B'E'$ and $C'F' \perp A'D'$, $BH \perp CF$ and $B'H' \perp C'F'$.

Proof: $\triangle BAE = \triangle B'A'E'$, and $\triangle CDF = \triangle C'D'F'$ } Right $\triangle$ with hypotenuse and acute $\angle$ equal.

$\therefore BE = B'E'$, $CF = C'F'$, and $CH = C'H'$,

$\therefore \triangle BHC = \triangle B'H'C'$, $\therefore BH = B'H'$,

$\therefore BHFE = B'H'F'E'$,

$\therefore BAE + CDF + BHC + BHFE = B'A'E' + C'D'F' + B'H'C' + B'H'F'E'$, or, as the whole equals the sum of its parts,

$$ABCD = A'B'C'D'.$$

That an assumption has been made in this proof that something that seems to happen in the figure necessarily happens may be seen by a simple method of showing the falsity of the conclusion. The following construction due to Dr. R. L. Moore will show that two quadrilaterals satisfying the hypothesis may be constructed as unequal as we like.

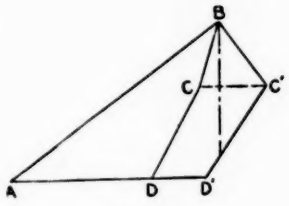

FIG. 5.

Produce a line segment AD making it any greater length, AD'. Draw angles ADC and $AD'C'$ equal. Take DC and $D'C'$ equal and take B on the perpendicular bisector of CC'. The quadrilaterals $ABCD$ and $ABC'D'$ satisfy the hypothesis without being equal.

The last instance will illustrate the fact that geometry knows no necessity except the logical and will commend to the teacher the advisability of putting the other two on logical foundations. But how? Shall we complicate the machinery by deducing the second from a postulate underlying the first or shall we have a

postulate for each. Geometry, wincing under a thrust at one of its real charms, cries out for the former; but pedagogy, unlike geometry, has other necessities than the logical, and perhaps classroom experience will dictate a compromise. In this we will agree, that the teacher who substitutes in place of a rational process the observation that certain lines seem to occupy certain positions in a drawn figure is not at that point teaching mathematics. It may be noted in passing that the assumption of Figs. 1 and 2 are neither essentially true nor essentially false, for falsity is a matter of inconsistence and until a fundamental such as Pasch's assumption is introduced, there is nothing with which they can be inconsistent.

True it is that the question is often raised if real geometry can actually be taught to the student in the high school. A society of mathematics teachers like ours, with a high regard for the real beauty of the science, with a keen appreciation of its cultural aspect, will hesitate to deny it. No teacher will answer in the negative until he has familiarized himself with a geometry foundation, consistent but irreducible, such as that of Hilbert or Veblen. Hilbert himself says that geometry made more nearly rational is thereby made more simple. Halsted—and it seems a pity that the name of a man with so ardent a love for pure geometry should not have appeared in the bibliography of science teaching published during the current year for the American Federation of Teachers of the Mathematical and the Natural Sciences, even if only to make intelligible to the reader the answer made to him by Professor Dewey—Halsted seems to believe that rational geometry can be taught to the high school student. We will perhaps hold such a logical system as purely an ideal toward which we will endeavor to make steady advance. Such men cannot be ignored and if we are at any point to teach something else for geometry these men must be answered. The really important consideration seems to be that we make the effort to approach the standard set by the geometer.

And why? Because, for one reason, the opportunity to teach geometry at all is at stake. But why should geometry be taught at all? We may answer: because of the beautiful deductive system it puts before the student; because of the stimulus this system furnishes to the systematizing of bodies of knowledge in

other fields. We must be in a position to answer: because of the power the student acquires in framing hypotheses, in sizing up situations, and in deducing conclusions. These things the student must be asked to do throughout the course. These things he must do in life; or rather we would say, the potentiality for training, to a greater or less degree in these directions forms a material part of his birthright.

Says Clifford: "This book (Euclid) has been for nearly twenty-two centuries the encouragement and guide of that scientific thought which is one thing with the progress of man from a worse to a better state. The encouragement; for it contained a body of knowledge that was really known and could be relied on. The guide; for the aim of every student of every subject was to bring his knowledge of that subject into a form as perfect as that which geometry had attained."

A list of concomitant benefits of the study of geometry of considerable length might be given, including such as skill in mechanical drawing, development of the space intuition, of accuracy in computation, and the application to physical space; but we are considering those benefits that have their source in the nature of the subject itself and we mention only these: the appreciation of a type of the beautiful and the useful, the stimulus to follow the model and the ability so to do.

Geometry teaching suffers from the fact that the subject seems to have a practical application. When a teacher becomes weary of the great task of assisting pupils through this system of deductive reasoning he is exposed to the danger of forsaking his real work for that of some practical application. He will say, perhaps, that his pupil is going to enter some trade and is in most need of specialized manual skill. What a pity! For if one student more than another needs to be able to think it is the one whose environment is in the greatest need of improvement.

Not long since the writer was privileged to hear a discussion of the advisability of establishing a vocational high school, led by a manufacturer and a journeyman tradesman. The manufacturer dwelt on the necessity of every man earning a living, of the necessity of getting recruits for the trades, and the advantage resulting from the co-operation between the factory

(*To be continued.*)

NEW BOOKS.

Differential and Integral Calculus. By LORRAIN S. HULBURT. New York: Longmans, Green, and Company. Pp. 481.

This book is intended for use in colleges and technical schools and is rather carefully written. It contains many excellent features which will appeal to the teacher and is one of the best books for the purpose that has come to our attention. It would seem, however, that too many writers on the calculus have aimed to meet the requirements of these two classes of students and in so doing have not entirely satisfied the desires of either. The teacher of the arts students will want to omit a good deal that the engineering students will want and *vice versa*. While the theory of the calculus is the same for all we believe it would be more satisfactory under present conditions to have a separate book for each class of students.

Elements of Trigonometry with Tables. By DANIEL A. MURRAY. New York: Longmans, Green, and Company. Pp. 231. $1.00 net.

In some respects this book is an abridgment of the author's "Elements of Trigonometry" with some rearrangement of chapters. For most teachers we believe it will be found an improvement on the earlier book, though the increased use of the unit circle and line functions is not desirable. Some teachers, however, may desire this increased use of line functions and for them it will be an improvement. It would appear to be a very teachable book for those who desire to begin with a treatment of acute angles.

Elements of Applied Mathematics. By HERBERT E. COBB. Boston: Ginn & Company. Pp. 274. $1.00.

This book is constructed on the principle that arithmetic, algebra, and geometry are closely linked together. The work outlined consists largely of lists of problems based on the student's preceding work in mathematics, illustrating the work in the shops and laboratories, and of simple experiments and exercises in the mathematics classroom, where the pupil, by measuring and weighing, secures his own data for numerical computations and geometrical constructions.

The College Engineering Notebook. By ROBERT E. MORITZ. Boston: Ginn & Company. $1.00.

This notebook is designed for the use of students in civil, mechanical, and electrical engineering. It contains ninety sheets of high-grade rectangular-co-ordinate paper, five sheets of polar-co-ordinate paper, and five sheets of logarithmic-co-ordinate paper. The reverse side of each sheet is ruled horizontally with vertical crosslines at intervals of two and one half centimeters. This arrangement adapts each sheet

equally well for computation purposes, for the construction of graphs, and for ordinary use.

Additional features of the book are lists of the most important mathematical formulas from algebra, geometry, trigonometry, analytics, and the calculus. These are followed by separate pages of carefully selected formulas from mechanics, surveying, strength of materials, mechanism, machine design, thermodynamics, electricity, and magnetism. The back part of the book contains a four-place table of logarithms, and short tables of natural logarithms, trigonometric functions, exponential functions, squares and square roots, cubes and cube roots, reciprocals, and hyperbolic functions. Finally, it contains eight sets of type curves.

Complete School Algebra. By H. E. HAWKES, W. A. LUBY, F. C. TOUTON. Boston: Ginn and Company. Pp. 507. $1.25.

The "Complete School Algebra," which includes between the covers of a single volume—with the necessary adaptation and abridgment—all the material of the authors' "First Course in Algebra" and "Second Course in Algebra," is designed for those schools which find a one-book course best suited to their needs.

The first twenty-three chapters contain the greater portion of the work usually taken up during the first year. Then follows the review material, each topic being given a broader and more advanced treatment than is permissible in first-year work. New matter is used throughout, and many new applications are given in order to make a fresh and inviting appeal to the student. In the remaining chapters those advanced topics considered necessary by the best secondary schools are included.

First Principles of Algebra. By H. E. SLAUGHT and N. J. LENNES. Boston: Allyn and Bacon. Pp. 280.

The authors in writing this book have kept two aims before them: (1) To provide a gradual and natural introduction to the symbols and processes of algebra. (2) To give vital purpose to the study of algebra by using it to do interesting and valuable things. The equation is introduced and developed early, and the principles are codified in a few short rules. It is an interesting and carefully written book.

First Year Algebra. By WEBSTER WELLS and WALTER W. HART. Boston: D. C. Heath and Company. Pp. 340. 90 cents.

This book shows some new blood has been infused into the older editions of Wells's book. It contains several excellent features and a student who had carefully completed it would have a good introduction to algebra.

Durell's School Algebra. By FLETCHER DURELL. New York: Charles E. Merill Co. Pp. 507. $1.10.

This is an attractive and interesting book. Its pages are well spaced and give an effect of simplicity to the text.

The introduction of formal algebra is gradual and the order seems teachable. One of the best features from the standpoint of pedagogy is the handling of exponents and radicals, their relation being used to advantage.

There are, as is usually the case, some features to which exception might be taken; for example, the failure to classify the cases of factoring under as few heads as possible, the emphasis given the *ux* method of solving simultaneous equations, rather than to the method by elimination of the independent term, etc.

Among other features worthy of mention are the historical material, the sets of review examples, the placing in the appendix of topics of doubtful value in an elementary course, and the introduction of material for problems, the pupils being required to make their own questions.

Plane Trigonometry and Tables. By FLETCHER DURELL. New York: Charles E. Merrill Co. Pp. 298. $1.25.

This is an excellent trigonometry, both in subject matter and in arrangement. One of its strong features is the way in which the subject is made to appear reasonable by a clear exposition of the advantages of certain of its methods and formulas.

The tables are complete, containing tables for decimal parts of a degree as well as for minutes, and they are easy to the eye.

The methods used are, as a whole, modern and clearly set forth. The arrangement of logarithmic work is good, and the method of indicating the given and required parts of triangles is simple and effective.

In so strong a book, it seems a pity that the author should have overlooked so simple a generalization as should be used in the proof of the Law of Lines. It seems as if authors of trigonometry fix their minds so firmly on function of acute angles that they overlook the possibilities of angles of more than ninety degrees.

The Francis W. Parker School Year Book. Vol. I. By the FACULTY OF THE FRANCIS W. PARKER SCHOOL. Chicago. Pp. 140. Price 35c. Press of the Francis W. Parker School, 330 Webster Avenue, Chicago, 1912.

The faculty of the Francis W. Parker School, Chicago, has undertaken to publish, annually, a volume, entitled *The Francis W. Parker School Year Book*, consisting of a series of concrete illustrated reports of work actually done in the school.

This school is in the nature of an educational laboratory, or an experiment station in education. Being unhampered by the traditions that beset the average school, and free to experiment in any way with the content of the course of study and methods of organization and teaching, the school is endeavoring to carry out certain modern fundamental principles and aims in education—principles and aims advanced especially by Colonel Francis W. Parker, one of the pioneers in the modern educational movements in this country.

The school proposes to share, with the general educational public, the benefits of its experimentation, through the Year Book. Each volume will be devoted to some one phase of education as actually worked out in the school.

Volume I. is devoted to the *social motive in school work*, and consists of a number of reports dealing with phases of handwork, music, dramatics, etc., in which the *social motive* predominates. The reports are plentifully illustrated—thirty-six illustrations in all—and show in a concrete way how the projects or activities described are made use of in the class room. The experiences of the school are thus suggested in dealing with the great educational problems of motivation, correlation, etc., of the present day. The volume should be of immense practical value to every teacher in the country.

Work and Play With Numbers. By GEORGE WENTWORTH and DAVID EUGENE SMITH. Boston: Ginn and Company. Pp. 144.

NOTES AND NEWS.

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